nelsonnet

Name: _____

TOPIC TEST

Arithmetic sequences and series

- Time allowed: 45 minutes
- Part A: 10 multiple-choice questions (10 marks)
- Part B: 12 free-response questions (40 marks)
- Total: 50 marks

Part A

10 multiple-choice questions
1 mark each: 10 marks
Circle the correct answer.

1 Which of the following sequences is arithmetic?

  A 2, 4, 8, 16, …

  B 2, 3, 5, 8, …

  C 2, −2, −6, −10, …

  D 2, −2, 4, −8, …

2 Find the value of x in the arithmetic sequence $2\frac{1}{2}, 1\frac{1}{3}, x, -1, \ldots$

  A $-\dfrac{1}{2}$

  B $\dfrac{1}{6}$

  C $\dfrac{1}{3}$

  D $\dfrac{1}{2}$

3 Given $t_n = 3n + 2$, the 10th term of this sequence is:

  A 5

  B 10

  C 17

  D 32

4 Find the nth term of the sequence 7, 11, 15, …

  A $t_n = n + 7$

  B $t_n = 3n + 7$

  C $t_n = 4n + 7$

  D $t_n = 4n + 3$

5 Find the 50th term of the sequence 7, 11, 15, …

   A 54

   B 193

   C 203

   D 354

6 The first term of an arithmetic sequence is -8 and its 5th term is 16.

Find the common difference in this sequence.

   A -8

   B 2

   C 6

   D 8

7 Find the first positive term of the arithmetic sequence $-40, -34, -28, …$

   A 1

   B 2

   C 4

   D 6

8 The 3rd term of an arithmetic sequence is 5 and its 4th term is 2.

Find the 20th term in this sequence.

   A -40

   B -43

   C -46

   D -49

9 Find the sum of the first 20 terms of the sequence $\sqrt{3} + 3\sqrt{3} + 5\sqrt{3} + \cdots$

   A $400\sqrt{3}$

   B $40\sqrt{3}$

   C $39\sqrt{3}$

   D $20\sqrt{3}$

10 Evaluate $\displaystyle\sum_{5}^{30} 2\pi n + 3$.

   A $910\pi + 78$

   B $875\pi + 75$

   C $70\pi + 6$

   D $26\pi + 39$

Part B

12 free-response questions
40 marks
Show your working where appropriate.

11 For the series −12, −9, −6, −3, …

    a what is its common difference?

_____

    b find its nth term and hence the 50th term.

_____

_____

_____

_____

[3 marks]

12 Given the nth term of a series is $t_n = 5n + 2$, which term is equal to 197?

_____

_____

_____

_____

[2 marks]

13 What is the 100th term of the series $7.4 + 11.2 + 15 + 18.8 + \ldots$?

_____

_____

_____

_____

[3 marks]

14 The 7th term of an arithmetic series is 14 and the 10th term is 23.

    What is the 30th term?

_____

_____

_____

_____

[4 marks]

15 Evaluate the series:

$$6.1 + 9.9 + 13.7 + 17.5 + \ldots + 116.3$$

[3 marks]

16 Simplify the sum of $2x + 3y$, $3x + 2y$, $4x + y$, … to 30 terms.

[3 marks]

17 For the sequence 64, 52, 40, 28, … find:

a the 101st term

b the sum of the first 101 terms

[4 marks]

18 a Show that $x + 3x + 5x + \dots$ is an arithmetic series.

_____

_____

_____

_____

b What is the 25th term of this series?

_____

_____

_____

_____

c If $x = 4$, evaluate the 25th term.

_____

_____

_____

_____

d Find the sum of the first 25 terms if $x = 4$.

_____

_____

_____

_____

[4 marks]

19 How many terms does it take for the series $1 + 3 + 5 + 7 + \dots$ to first exceed a sum of 500?

_____

_____

_____

_____

[3 marks]

20 The 5th term of an arithmetic series is 10 and the sum of the first 10 terms is 70.

What is the sum of 20 terms?

[4 marks]

21 An 8 cm seedling grows by 6 mm in the first week it is planted. It then keeps growing by 6 mm each week until it reaches its full height after 12 weeks.

    a After how many weeks will the plant be 11 cm tall?

    b How tall is this plant after 12 weeks?

[2 marks]

22 A bricklayer has 10 days to lay a brick wall needing a total of 1100 bricks.

If he decides to start with k bricks the first day and then increases the number of bricks each day by k, find the value of k so that he finishes building the entire wall in 10 days.

[3 marks]

23 Prove that $t_n = S_n - S_{n-1}$.

_____

_____

_____

_____

[2 marks]

This is the end of the test.
Use this page and the back for extra working space.

Answers

1 C	**2** B	**3** D	**4** D
5 C	**6** C	**7** B	**8** C
9 A	**10** A		

11 a $d = 3$

b $t_n = 3n - 15$, $t_{50} = 135$

12 t_{39}

13 383.6

14 83

15 1836

16 $495x - 345y$

17 a -1136

b $-54\ 136$

18 a $d = 2x$

b $49x$

c 196

d 2500

19 23

20 -460

21 a 6

b 14.6 cm

22 $k = 20$

23 Prove that $t_n = S_n - S_{n-1}$.

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\big[2a + (n-1)d\big]$$

$$S_{n-1} = \frac{n-1}{2}\big(2a + [(n-1)-1]d\big)$$

$$= \left(\frac{n-1}{2}\right)\big[2a + (n-2)d\big]$$

$$S_n - S_{n-1} = \frac{n}{2}\big[2a + (n-1)d\big] - \left[\left(\frac{n-1}{2}\right)\big[2a + (n-2)d\big]\right]$$

$$= an + \frac{n(n-1)d}{2} - \left[a(n-1) + \frac{(n-1)(n-2)d}{2}\right]$$

$$= an + \frac{n(n-1)d}{2} - an + a - \frac{(n-1)(n-2)d}{2}$$

$$= a + \frac{n(n-1)d}{2} - \frac{(n-1)(n-2)d}{2}$$

$$= a + \frac{(n-1)d}{2}\big[n - (n-2)\big]$$

$$= a + \frac{(n-1)d}{2}\big[2\big]$$

$$= a + (n-1)d$$

$$= t_n$$