Preliminary work.

This book assumes that you are already familiar with a number of mathematical ideas from your mathematical studies in earlier years.

This section outlines the ideas which are of particular relevance to Unit One of the *Mathematics Applications* course and for which some familiarity will be assumed, or for which the brief explanation given here may be sufficient to bring your understanding of the concept up to the necessary level.

Read this "preliminary work" section and if anything is not familiar to you, and you don't understand the brief explanation given here, you may need to do some further reading to bring your understanding of those concepts up to an appropriate level for this unit. (If you do understand the work but feel somewhat "rusty" with regards to applying the ideas some of the chapters afford further opportunities for revision, as do some of the questions in the miscellaneous exercises at the end of chapters.)

☞ Chapters in this book will continue some of the topics from this preliminary work by building on the assumed familiarity with the work.

☞ The miscellaneous exercises that feature at the end of each chapter may include questions requiring an understanding of the topics briefly explained here.

- **Types of number.**

  It is assumed that you are already familiar with:

  Counting numbers: 1, 2, 3, 4, 5, 6, 7, ...

  Whole numbers: 0, 1, 2, 3, 4, 5, 6, 7, ...

  Integers: ... −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, 7, ...

  It is also anticipated that you are familiar with *fractions* and *decimals*, including negatives, that you can add, subtract, multiply and divide such numbers (with a calculator when appropriate) and are able to convert between these forms of representation.

- **Powers.**

  You should already be familiar with the idea of raising a number to some *power* (perhaps squared, cubed etc), the idea of the *square root* or *cube root* of a number and be familiar with zero and negative integers as powers.

  For example $5^2 = 5 \times 5 = 25$ $\quad 6^3 = 6 \times 6 \times 6 = 216$ $\quad 2^0 = 1$ $\quad 3^0 = 1$

  $\sqrt{25} = 5$ $\quad \sqrt[3]{216} = 6$ $\quad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $\quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

  You are probably also familiar with numbers expressed using *standard form* or *scientific notation*, e.g. $260\,000 = 2{\cdot}6 \times 10^5$, $\quad 53\,200\,000 = 5{\cdot}32 \times 10^7$,

  $0{\cdot}000\,042 = 4{\cdot}2 \times 10^{-5}$, $\quad 0{\cdot}0015 = 1{\cdot}5 \times 10^{-3}$.

- **Rule of order.**
  An ability to correctly apply the *rule of order* is assumed. For example, when evaluating $2 + 3^2$ you should know to square the 3 first and then add the 2:

$$\begin{aligned} 2 + 3^2 &= 2 + 9 \\ &= 11 \end{aligned}$$

  This rule of order is sometimes remembered as BIMDAS:

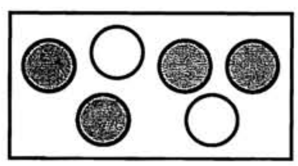
  Brackets
  Indices
  Multiplication and **D**ivision in the order they occur
  Addition and **S**ubtraction in the order they occur

- **Rounding and truncating.**
  Answers to calculations may need *rounding* to a suitable or specified accuracy. For example:

$193{\cdot}3 \div 17 =$	11·370588	if we round our answer to six decimal places,
	11·37059	if we round our answer to five decimal places,
	11·3706	if we round our answer to four decimal places,
	11·371	if we round our answer to three decimal places,
	11·37	if we round our answer to two decimal places,
	11·4	if we round our answer to one decimal place,
	11	if we round our answer to the nearest integer.

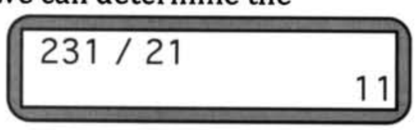
  In some cases the situation may make *truncating* more appropriate than rounding. Suppose for example we have \$10 and wish to buy as many chocolate bars costing \$2·15 each as possible. Whilst \$10 ÷ \$2·15 is 4·65 if we round to two decimal places, 4·7 if we round to one decimal place and 5 if we round to the nearest integer, a more appropriate answer is obtained by truncating to 4 as that is the number of chocolate bars we would be able to buy with our \$10 (and we would have \$1·40 change). If we truncate to an integer we discard the decimal part entirely.
  It is assumed that you can round appropriately in order to obtain an approximate answer to a calculation (i.e. you can *estimate* the answer).

- **Ratios.**
  The idea of comparing two or more quantities as a *ratio* should be something you are familiar with.



  For example, for the diagram on the right the ratio of unshaded circles to shaded circles is $2 : 4$
  which simplifies to $1 : 2$

  Suppose the ratio of males to females in a school is $17 : 21$.
  If we know that there are 231 females in the school we can determine the number of males.



$$\begin{aligned} \text{Males : females} &= 17 : 21 \quad \times 11 \\ &= \; ? : 231 \end{aligned}$$

  The number of males = 17 × 11, i.e. 187.

- **Rates.**

A rate shows how one quantity changes with relation to another. For example how the total number of items produced may increase for each extra hour of production, how the total cost changes with each extra kilogram we purchase, how the total distance we have travelled increases with each extra second we travel etc.

Some other examples of rates used in everyday life are given below.

Cost of fuel. cents / litre	Cost of meat. $ / kg	Postal rates. $ / kg
Pay rate. $ / hour	Pulse. beats / minute	Infection rate. cases / year
Download rate. bytes / second	Speed. km / hour	Typing rate. words / minute
Density. g / cm^3	Sports rates. points / game	Sports rates. runs / over

In its simplest form a rate is usually expressed as "per 1 unit".
Thus a rate of $60 / 2 kg would usually be written as $30 / kg.
31 heart beats / half minute would usually be written as 62 beats / minute.
However rates are not always expressed in this simplest form. For example fuel consumption is sometimes given as litres / 100 km and when rate involves the population of a country some rates are given as "per 1000 of population", as in birth rates, death rates, etc.

A rate often involves a comparison between two quantities that have different units of measure. For example kilometres per hour, dollars per kilogram etc. However the word rate is also frequently used when two quantities involved in the rate have the same unit. The rate may then be given as a percentage rate. Some examples are given below.

Tax rate. e.g. 10% goods and services tax (GST) e.g. 30% income tax	Rate of commission. e.g. $20 commission for each $500 of sales. i.e. 4% commission.
Conviction rate. e.g. 38 cases in each 100 result in a conviction. i.e. 38% conviction rate.	Unemployment rate. e.g. 58 unemployed people per 1000 possibles. i.e. 5·8% unemployment
Annual death rate. e.g. 16 deaths per 1000 of population. i.e. 1·6% death rate.	Insurance rate. e.g. $3·50 per $1000 insured. i.e. 0·35% of value.

- **Percentages.**
  It is assumed that you know, or can determine, the common equivalences between decimals, fractions and *percentage*. Some of these are shown below:

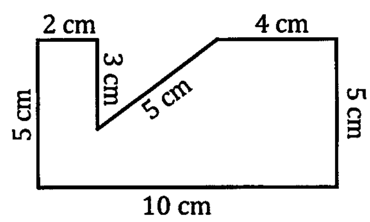
Decimal	Fraction	Percentage
0·01	$\frac{1}{100}$	1%
0·1	$\frac{1}{10}$	10%
0·2	$\frac{2}{10}$ i.e. $\frac{1}{5}$	20%
0·25	$\frac{25}{100}$ i.e. $\frac{1}{4}$	25%
0·5	$\frac{5}{10}$ i.e. $\frac{1}{2}$	50%
0·75	$\frac{75}{100}$ i.e. $\frac{3}{4}$	75%
0·8	$\frac{8}{10}$ i.e. $\frac{4}{5}$	80%
1·1	$1\frac{1}{10}$	110%

  Whilst it is also assumed that you would have previously encountered the ideas of *expressing some amount as a percentage of a total amount* and of *finding percentages of amounts* these skills will be revised and further practiced in the chapter on percentages.

- **Understanding formulae.**
  The rule $C = 2 \times \pi \times r$ tells us how we can determine C, the circumference of a circle, knowing π and r, the radius of the circle. It is assumed that you are already familiar with the convention of leaving out the multiplication sign and writing the formula simply as $C = 2\pi r$.

Similarly	$A = b \times h$	is written	$A = bh$
	$V = I \times R$	is written	$V = IR$
	$v = u + a \times t$	is written	$v = u + at$
	$A = \pi \times r \times r$	is written	$A = \pi r^2$

- **Perimeter.**
  The perimeter of a closed shape is the distance all around the outside. It is the length of the outline of the shape. (For circles the perimeter is known as the *circumference*.)
  For the shape shown on the right the perimeter is
  34 cm.
  (= 10 cm + 5 cm + 4 cm + 5 cm + 3 cm + 2 cm + 5 cm)

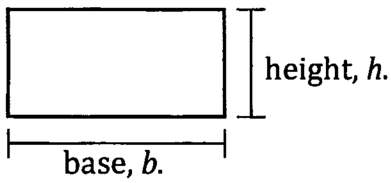


- **Area.**

  Chapter 8 involves finding perimeters and areas of various shapes. The chapter assumes you are already familiar with determining the area of rectangles, triangles, parallelograms, trapeziums and circles and with determining the perimeters of such shapes, given sufficient information.

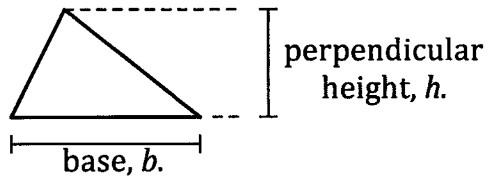
  The formulae for the areas of these shapes, and for the circumference of a circle are given below by way of a reminder.

Rectangle



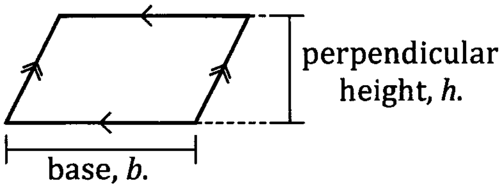
Area = $\boldsymbol{b \times h}$

Triangle



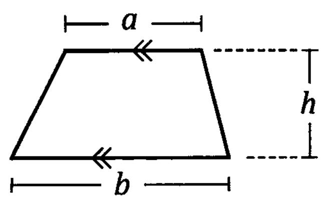
Area = $\boldsymbol{\dfrac{b \times h}{2}}$

Parallelogram



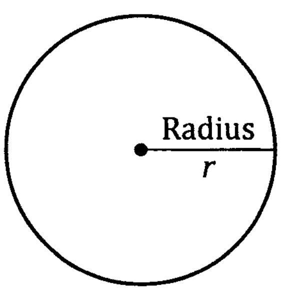
Area = $\boldsymbol{b \times h}$

Trapezium



Area = $\boldsymbol{\dfrac{a + b}{2} \times h}$

Circle, of radius r.



Area = $\boldsymbol{\pi r^2}$

Circumference = $\boldsymbol{2\pi r}$
(or $\boldsymbol{\pi d}$ where $\boldsymbol{d}$ is the diameter)

- **Units of length, area, volume and capacity.**
  It is assumed that you are familiar with the following metric units.

Length

One millimetre	1 mm		
One centimetre	1 cm	=	10 mm
One metre	1 m	=	100 cm
One kilometre	1 km	=	1000 m

Area

(The size of the surface that a two dimensional shape occupies.)

One square millimetre	1 mm^2	=	1 mm × 1 mm
One square centimetre	1 cm^2	=	1 cm × 1 cm
One square metre	1 m^2	=	1 m × 1 m
One hectare	1 ha	=	100 m × 100 m
One square kilometre	1 km^2	=	1 km × 1 km

Volume

(The amount of space a three dimensional object occupies).

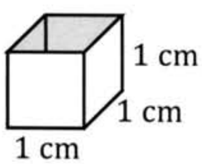
One cubic millimetre	1 mm^3	=	1 mm × 1 mm × 1 mm
One cubic centimetre	1 cm^3	=	1 cm × 1 cm × 1 cm
One cubic metre	1 m^3	=	1 m × 1 m × 1 m

Capacity

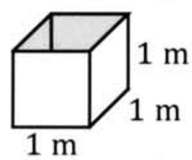
(The amount a container can hold.)

One millilitre	One litre	One kilolitre
1 mL	1 L	1 kL
	= 1000 mL	= 1000 L

The container shown below has a capacity of 1 mL.



The container shown below has a capacity of 1 kL.



- **Use of technology.**
  Students following this unit may come to the course with very varied abilities in the use of graphic calculators and computers. Whatever your initial ability you are encouraged to make use of such technology whenever appropriate.
  If you are not familiar with this technology at the start of this course this does not need to be a concern but, as the course progresses, do try to become familiar with entering data into the columns of a spreadsheet on a computer or calculator and with carrying out straightforward operations on those entries such as finding the total of a list of numbers etc.